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THEORETICAL METHODS IN SHIP MANEUVERING

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ABSTRACT

The classical theory of ship maneuvering is based on the differential equations of motion which apply to the irrotational flow past a rigid body in an ideal fluid. To these are added semi-empirical corrections to account for viscous, free-surface and lifting-surface effects. Most theoretical models neglect viscous and free-surface effects, and treat lifting phenomena under the assumptions that the ship hull is slender, and that the lateral motions are small by comparison to the forward velocity.

Theoretical methods are especially useful in *shallow* or restricted water, and in the related situation where two vessels are in close proximity. Recent results based on the slender-body approach are reviewed, and compared with experiments. Suggestions for future research are outlined.

INTRODUCTION

Comprehensive surveys of ship maneuvering in deep and shallow water have been provided by several authors, including Norrbin (1970) and Motora (1972). The present lecture has the more limited objective of reviewing contributions which are based on the methods of theoretical hydrodynamics. The mathematical theory which forms the basis for experimental work in this field is covered separately, in the lecture to follow by Dr. Sharma.

Most theoretical analyses of ship maneuvering neglect viscous stresses in the fluid, and wave effects on the free surface. The ship hull is regarded as a rigid body, with six degrees of freedom. Equations of motion are derived by equating the external hydrodynamic pressure force and moment to the inertial force and moment associated with the ship's mass.

The classical approach to this type of problem is described by Lamb (1932). The motion of the fluid is assumed irrotational, and the resulting equations of motion for the ship involve linear functions of its acceleration and quadratic functions of its velocity. The coefficients of these equations are constants known as the added mass and added moment of inertia, or simple combinations thereof. Neglected in this approach are various hydrodynamic factors:

1. Lifting effects
2. Free-surface effects
3. Viscous stresses and separation
4. Propeller-hull-rudder interactions
5. Shallow-water effects
6. Restricted-water effects

Lifting effects must be accounted for in the simplest relevant model. The lateral motions introduce an effective angle of attack of the hull, relative to the incoming flow. In the steady state this is the "drift angle". As in the analogous problems of hydrofoils, propellers and other lifting surfaces, the movement of the ship through the fluid at a small angle of attack can be described by a suitable inviscid model, irrotational except for a thin sheet of shed vorticity trailing downstream from the stern.

The nature of the ship's "trailing edge" is of paramount importance. If the rudder and deadwood are regarded as an abrupt vertical trailing edge, of semi-span equal to the draft, and if the hull is assumed to be an elongated slender body, expressions can be derived for the lift force and moment. For deep water these confirm the algebraic form of the traditional mathematical model, adding to the classical "added-mass theory" lifting effects proportional to the instantaneous sway and yaw velocities.

Free-surface effects are important in other fields of ship hydrodynamics, notably as the basis of wave resistance in steady motion and of oscillatory motions in the vertical plane. The free surface is less important for ship maneuvers in the horizontal plane, due to the longer characteristic time scale relevant to these motions. With inertial effects at the free surface dominated by gravity, the corresponding boundary condition of zero vertical velocity is satisfied by adding an image to the submerged portion of the hull. The resulting "double body" is symmetrical about the plane of the free surface, and may be analysed without further consideration of this boundary. More complete treatments of free-surface effects by Hu (1961) and Chapman (1975) confirm that for low and moderate Froude numbers these effects are not of primary significance.

Viscosity may affect the flow field in two possible ways. In the boundary layer close to the hull the fluid velocity is modified substantially by viscosity. This boundary layer is of direct importance in the context of ship resistance in steady forward motion. More important in maneuvering is the possibility of a separated flow region on the low-pressure side of the hull which, if it occurs, will produce a "cross-flow drag" force on the hull. This is an important nonlinear contribution to the steady force and moment at large drift angles. In this context separation is analogous to the stall of high-aspect-ratio lifting surfaces. But unlike the latter phenomenon, which decreases the lift force from its linear value, the cross-flow drag on a slender body increases the lift or side force. This increment can be estimated as the product of a suitable two-dimensional drag coefficient and the square of the local lateral velocity.

Separation at large drift angles is accounted for in a semi-empirical manner by the Bollay low-aspect-ratio model, which has been applied to ship maneuvering by Inoue (1956), and recently by Inoue and Kijima (1978). This approach is useful if the drift angle is large compared to the slenderness or aspect ratio, but inconsistent with the linear theory for small drift angles as noted by Newman (1972). A more fundamental description of vortex shedding is outlined by Brard (1976) but this has not been reduced to a computational procedure.

Viscous separation at the stern is a more nebulous complication, of importance to the extent that it affects the rudder and propeller. The theoretician is most comfortable in assuming that the effects of viscous retardation at the stern are offset by the slipstream of the propeller, in which case both complications can be neglected. A semi-empirical analysis of propeller-hull-rudder interactions is described by Hess (1977).

In shallow water, the force and moment acting on the ship are affected significantly by the presence of the bottom. This is true even for steady forward motion, where the resulting change in the dynamic pressure causes a sinkage and trim, or "squat". Grounding may result in extreme cases. Mariners have empirical estimates for this effect, but a more rational approach has followed from the slender-body analysis of Tuck (1966) and from various extensions reviewed by Tuck (1978) and Beck (1979). Tuck's original calculations compare favorably with experiments, as shown in Figure 1. The principal exception, of limited practical importance, is in the vicinity of the critical Froude number based on depth; a nonlinear analysis of

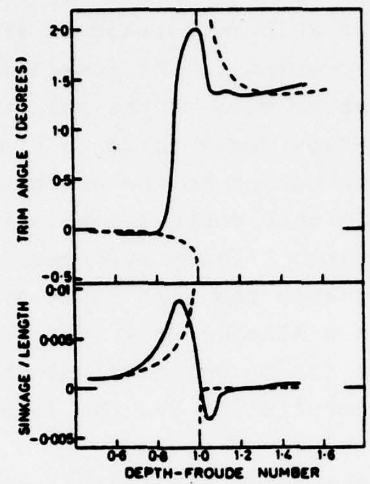

Figure 1. Sinkage and trim in shallow water, as functions of the Froude number $u/(gh)^{1/2}$. Solid curves are experimental results, and the dashed curves are computed from slender-body theory. (Reproduced from Tuck, 1978).

this regime is given by Lea and Feldman (1972).

Shallow-water effects are of particular importance for lateral ship maneuvers. Generally, the hydrodynamic force and moment are increased by the effect of the bottom. An important concept is that the bottom and its image above the free surface form a vertical "channel" to constrain the flow in horizontal planes. This effect increases with decreasing clearance beneath the keel. For zero clearance the flow past a maneuvering ship is essentially identical to that for a two-dimensional vertical strut or symmetrical wing section.

In practice a nonzero bottom clearance exists, and the flow is a combination of the two-dimensional lifting field with a superposed cross-flow "through" the strut. This cross-flow is proportional to the pressure difference between the two sides of the hull, and the resulting problem corresponds physically to two-dimensional flow past a porous cambered thin wing. A procedure for analysing this intermediate regime is developed by Newman (1969). Computations reveal a substantial relief in the cross-flow pressure gradient, even if the underkeel clearance is small compared to the draft.

The characteristic time to approach a steady state is $t=u/\ell$, where u is the forward velocity of the ship and ℓ is the relevant length scale. The draft is the important length scale in deep water, where the hull is effectively slender. The corresponding characteristic time is small, and after a short period the flow is quasi-steady. By comparison, the limiting case of a two-dimensional thin wing implies the chord or ship length to be the relevant length scale in shallow water. Well known results for the force and moment on an unsteady two-dimensional thin wing imply that a distance of several ship lengths must be travelled before a steady state is achieved.

In unsteady two-dimensional wing theory the force and moment are expressed as convolution integrals over the previous time history of the motion. This is the simplest and most relevant example of "memory effects" in ship maneuvering. More generally, the same type of convolution integral in time is appropriate for all cases where the present state of the flow is affected by previous motions of the ship. Other possible causes of memory effects are wave motions on the free surface, and vorticity shed along the hull. In circumstances where these effects are significant, the traditional equations of motion which depend simply on the ship's instantaneous velocity and acceleration must be discarded in favor of more complicated convolution integrals.

In deep water the importance of memory effects is debatable. But memory effects are of unquestionable importance in shallow water. Practical evidence for this statement is the mariner's experience that ship maneuvers take place more slowly in shallow water.

Restricted-water effects are significant in special circumstances, involving navigation in a canal, channel, and among islands or similar obstructions. A monograph of theoretical and experimental work on these topics is given by Fujino (1976). The simplest relevant theoretical problem is that of steady motion parallel to a vertical canal bank or wall, which by the method of images is equivalent to the parallel motion of two identical ships abreast. Thus, from the mathematical standpoint, an analogy exists between restricted-water effects and ship-to-ship interactions. In general the latter problem includes unsteady relative motions between two ships, with time-varying interaction forces and moments which must be anticipated by the helmsman. Several recent theoretical studies have been made of these problems.

The following sections are intended to provide more detailed background and illustrative examples of recent theoretical contri-

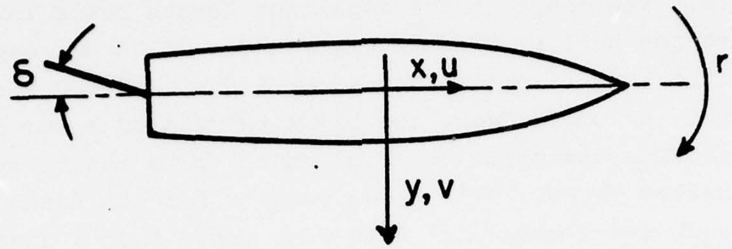

Figure 2. Definitions of coordinates, velocities, and rudder angle as viewed from the above.

butions, separated in the categories of deep, shallow and restricted water.

MANEUVERING IN DEEP WATER

In accordance with the standard notation described by Mandel (1967), Cartesian coordinates are defined with respect to the moving axes of the ship, as in Figure 2. The longitudinal x-axis is in the forward direction, y to starboard, and z downwards. The ship is assumed to have transverse symmetry about the plane y=0, and the origin is in the plane of the free surface. The longitudinal position of the origin will be chosen subsequently in the most convenient manner.

The following analysis is based on the use of slender-body approximations to describe the hydrodynamic forces on the hull. A similar and more detailed derivation by Newman (1977) employs a different coordinate system.

Our attention is restricted to unsteady motions of the ship in the horizontal plane (x,y), with translational velocity (u,v) and rotational velocity r. The longitudinal component u(t) often is assumed to have a constant value u_1, but the theoretical analysis to follow does not require this assumption (cf. Katz and Weihs, 1979). Under these circumstances the resultant hydrodynamic force acting on the cross-section of the hull is

$$f(x,t) = -\left(\frac{\partial}{\partial t} - u\frac{\partial}{\partial x}\right)[(v+rx)m'] . \tag{1}$$

This equation can be interpreted physically in terms of the time-rate-of-change of fluid momentum in a transverse plane. Here m'(x) denotes the two-dimensional added-mass coefficient of the submerged

transverse section of the ship, which depends predominantly on the local draft. The coefficient m' and the force f are defined in the two-dimensional sense, per unit length along the hull.

The three-dimensional sway force acting on the ship is

$$Y = \int f(x,t)dx$$
$$= -\int(\dot{v}+\dot{r}x)m'dx + u[(v+rx)m']_{Stern}^{Bow}, \tag{2}$$

where the integrals are over the submerged length of the ship. The yaw moment N can be treated in a similar manner, integrating by parts to obtain the result

$$N = \int xf(x,t)dx$$
$$= -\int(\dot{v}+\dot{r}x)m'xdx - u\int(v+rx)m'dx$$
$$+u[x(v+rx)m']_{Stern}^{Bow}. \tag{3}$$

The treatment of the integrated terms in (2) and (3) is based on the usual approach in low-aspect-ratio wing theory. The shape of the bow is such that m' vanishes at this point. At the stern ($x=x_s$) there is an abrupt trailing edge and nonvanishing $m'(x_s)$.

Since the velocity components (v,r) are constant with respect to the integrals in (2) and (3), the results can be expressed in the form

$$Y = Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_v v + Y_r r, \tag{4}$$

$$N = N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_v v + N_r r. \tag{5}$$

Here

$$Y_{\dot{v}} = -\int m'dx, \tag{6}$$

$$Y_{\dot{r}} = N_{\dot{v}} = -\int m'xdx, \tag{7}$$

$$N_{\dot{r}} = -\int m'x^2dx \tag{8}$$

are the (negative) added-mass coefficients, and

$$Y_v = -um'(x_s), \tag{9}$$

$$Y_r = -ux_s m'(x_s), \tag{10}$$

$$N_v = -ux_s m'(x_s) - u\int m'dx, \tag{11}$$

$$N_r = -ux_s^2 m'(x_s) - u\int m'xdx \tag{12}$$

are the force and moment coefficients due to steady sway and yaw velocities.

The added-mass coefficients (6-8) can be regarded as strip-theory approximations to the exact three-dimensional added mass and added moment of inertia. These components of the total force and moment can be deduced from a purely two-dimensional analysis of the flow at each transverse section. By comparison, the "steady" coefficients (9-12) result from longitudinal convection of momentum along the hull. The integrals in (11-12) represent the "Munk" moment due to the distribution of added mass along the length. The remaining contributions in (9-12) are due specifically to the existence of a trailing edge at the stern, and associated lifting effects.

This simplified description of the hydrodynamic force and moment is completed by adding the contributions due to a nonzero rudder angle δ. This angle is defined with the convention that $\delta>0$ results in a positive force in the y-direction during forward motion. For a conventional rudder this corresponds to movement of the rudder to port, and turning of the ship in the same direction. The resulting force and moment can be expressed in the linearized form

$$Y = Y_\delta \delta = m'(x_s)u^2\delta, \tag{13}$$

$$N = N_\delta \delta = x_s m'(x_s)u^2\delta. \tag{14}$$

The last results follow from slender-body theory in an analogous manner to (6-12), treating the rudder as a trailing-edge flap of the same span as the double body, and neglecting unsteady effects which are significant only over time scales characterized by the rudder chord length.

The total hydrodynamic force or moment is the sum of (13) or (14) and (4) or (5). Equating these to the inertial force and moment due to the ship's internal mass gives a pair of coupled first-order differential equations of motion, which can be arranged in the form

$$(m-Y_{\dot v})\dot v + (mx_G-Y_{\dot r})\dot r - Y_v v + (um-Y_r)r = Y_\delta\delta, \tag{15}$$

$$(mx_G-N_{\dot v})\dot v + (I_z-N_{\dot r})\dot r - N_v v + (umx_G-N_r)r = N_\delta\delta. \tag{16}$$

In these equations m is the ship's mass, centered at $x=x_G$, and I_z is the polar moment of inertia about the vertical axis.

The only terms in (15) and (16) which do not have obvious physical interpretations are the force umr and moment $umx_G r$, which account for the rate of change of the ship's momentum due to rotation of the non-inertial reference frame (x,y,z). There is a hydrodynamic

force of similar form, involving the longitudinal added mass, but for a streamlined body this is negligible compared to m.

The coupled equations (15-16) can be replaced by a second-order equation for r, of the general form

$$A\ddot{r}+B\dot{r}+Cr = D\dot{\delta}+E\delta . \qquad (17)$$

With the origin at $x_G=0$, the coefficients of this equation are given by

$$A = (m-Y_{\dot{v}})(I_z-N_{\dot{r}}) -N_{\dot{v}}Y_{\dot{r}} , \qquad (18)$$

$$B = N_{\dot{v}}(um-Y_r) - N_r(m-Y_{\dot{v}}) - Y_v(I_z-N_{\dot{r}}) - N_vY_{\dot{r}} , \qquad (19)$$

$$C = N_v(um-Y_r) + Y_vN_r , \qquad (20)$$

$$D = N_{\dot{v}}Y_\delta + N_\delta(m-Y_{\dot{v}}) , \qquad (21)$$

$$E = N_vY_\delta - Y_vN_\delta . \qquad (22)$$

With the time derivatives in (17) set equal to zero, one obtains the steady turning rate $r=E\delta/C$.

There are two fundamentally different options for determining the coefficients in these equations of motion. The usual approach is to regard (15-16) or (17) as a "mathematical model" and to use experimental data for the coefficients. Most appropriate in this context are captive-model tests where the motions of the model are prescribed, and the forces measured directly. Alternatively, the theory can be used in a self-contained manner, by using (6-14) with suitable values for the two-dimensional added-mass coefficient $m'(x)$.

To pursue the theoretical approach, numerical techniques can be used to determine m' for prescribed ship sections and their corresponding double-body image. Alternatively, these sections can be approximated by simple geometric shapes such as an ellipse, rectangle, or by the generalized Lewis forms described by Landweber and Macagno (1959). A reasonable approximation follows from the added-mass coefficient for an ellipse,

$$m' = \frac{\pi}{2}\rho T^2 , \qquad (23)$$

where ρ is the fluid density and T is the local draft. Refinements of this approximation are discussed by Berg and Utnes (1978). Generally speaking the added mass is increased relative to (23), if the sectional-area coefficient is greater than the value $\pi/4$ for an ellipse, and conversely.

	SLENDER-BODY THEORY (RECTANGULAR PLANFORM)		EXPERIMENTS
	GENERAL	MARINER($\times 10^3$)	MARINER($\times 10^3$)
Y_v	$-\frac{\pi}{2}\rho u T^2$	-7.1	-13.5±2.3
Y_r	$\frac{\pi}{4}\rho u T^2 L$	3.6	2.90±0.68
$Y_{\dot{v}}$	$-\frac{\pi}{2}\rho T^2 L$	-7.1	-6.85±0.75
$Y_{\dot{r}}$	0	0	0.27±0.07
Y_δ	$\frac{\pi}{2}\rho u^2 T^2$	7.1	2.76±0.63
N_v	$-\frac{\pi}{4}\rho u T^2 L$	-3.6	-3.66±0.54
N_r	$-\frac{\pi}{8}\rho u T^2 L^2$	-1.8	-2.17±0.52
$N_{\dot{v}}$	0	0	-0.20±0.07
$N_{\dot{r}}$	$-\frac{\pi}{24}\rho T^2 L^3$	-0.6	-0.39±0.10
N_δ	$-\frac{\pi}{4}\rho u^2 T^2 L$	-3.6	-1.31±0.27

Table 1. Theoretical and Experimental Values of the Coefficients in the Equations of Motion (15-16). The last two columns are nondimensionalized by $\frac{1}{2}\rho$, L, and u, and multiplied by a common factor 10^3. The last column shows the mean and standard deviation of experimental data tabulated by Motora (1972).

The simplest illustration of the theoretical approach follows by using (23) and assuming the hull to be of constant draft or rectangular planform. The resulting coefficients are tabulated in the first column of Table 1. These hydrodynamic coefficients are independent of the beam and fullness of the ship, as a result of using (23). The second column of Table 1 shows the corresponding theoretical coefficients, nondimensionalized in the usual manner with respect to $\frac{1}{2}\rho$, the length L, and the velocity u, and with length-draft ratio L/T=21, corresponding to the Mariner hull. The experimental values for the Mariner are summarized in the last column of Table 1, based on averaging the data from various sources which are reported by Motora (1972).

The agreement between theory and experiments can be judged by a comparison of the last two columns in Table 1. Coefficients which depend predominantly on the distribution of added mass along the length are fairly well predicted by the theory, i.e. the three-dimensional added-mass coefficients and the steady turning moments. The drift force Y_v is seriously underpredicted by theory; Norrbin (1970) discusses this point and notes the sensitivity of the experimental data to nonlinear effects. Similar comparisons are made for other ship forms by Tsakonas, et al (1977) and by Berg and Utnes (1978).

The greatest defect of the theory is in overpredicting the rudder force and moment. This is not surprising, since the rudder span is less than the total draft. Moreover, slender-body theory leads to the conclusion that the steady lift force depends exclusively on the slope at the trailing edge, and thus that the force due to the rudder is independent of its chord length. Since the chord length is comparable to the span, a more elaborate lifting-surface representation is appropriate. Hess (1978b) discusses this problem, and carries out a finite-aspect-ratio analysis, but the experimental comparison is disappointing. Hess concludes that rudder interactions with the propeller slipstream and wake preclude reliable theoretical predictions.

For short periods of time after an initial rudder angle is set, the solution of (15-16) or (17) can be determined by series-expansion in powers of t. This calculation is simplified by choosing the origin at a suitable point near the midship section, such that (in theory) $mx_G - N_{\dot{v}} = mx_G - Y_{\dot{r}} = 0$. With this choice, the solution for a step-function rudder angle at t=0 is

$$v = Y_\delta \delta t/(m-Y_{\dot{v}}) + O(t^2) \, , \tag{24}$$

$$r = N_\delta \delta t/(I-N_{\dot{r}}) + O(t^2) \, , \tag{25}$$

The transverse position y_o of the ship's centerplane, relative to a fixed coordinate system, is defined by the solution of

$$\left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right) y_o(x,t) = v+rx \, . \tag{26}$$

With ψ the yaw angle and $r=\dot{\psi}$, it follows that

$$y_o = \int_o^t (v+rx+u\psi)dt$$

$$= \frac{1}{2} t^2 \left[Y_\delta \delta/(m-Y_{\dot{v}}) + N_\delta \delta x/(I_z-N_{\dot{r}}) \right]$$

$$+O(t^2) \ . \qquad (27)$$

Thus the ship will pivot initially about the point

$$x_p = -\frac{Y_\delta(I_z-N_{\dot{r}})}{N_\delta(m-Y_{\dot{v}})} \ . \qquad (28)$$

Using the theoretical coefficients,

$$x_p \simeq \frac{I_z+\frac{\pi}{24}\rho T^2 L^3}{\frac{1}{2}L(m+\frac{\pi}{2}\rho T^2 L)} \ . \qquad (29)$$

Assuming further that the radius of gyration is equal to L/4, we obtain the estimate

$$x_p/L = \frac{\pi/6+\beta/4}{\pi+2\beta} \ , \qquad (30)$$

where

$$\beta = \frac{m}{\rho T^2 L} \qquad (31)$$

is the product of the beam-depth ratio and block coefficient. For typical ship hulls the pivot point is situated forward of the midship section a distance of about L/7. Thus, during the initial stage of a turn, the bow will turn inward and the stern outward. The midship point also will turn outward slightly, by an amount which is exaggerated in many illustrations of trajectories, such as those shown in Mandel (1967) and Newman (1977). The maximum outward trajectory of the midship point can be estimated by retaining terms of order t^2 in (24-25) and $O(t^3)$ in (27). For a typical ship hull the result of this calculation is an outward displacement amidships of magnitude less than L/100.

The differential equations (15-16) or (17) can be solved in general by Laplace transform techniques. Of particular importance is the question of stability, which depends on the characteristic roots of the quadratic equation with coefficients (A,B,C). It is easily shown that A and B are positive, and the solutions are stable or unstable according as C is positive or negative respectively.

The theoretical coefficients are qualitatively useful here, in predicting the sign of the coefficient C to be the same as the factor $\pi-2\beta$. Thus full ships are unstable, and conversely, with a neutral point in the range of practical values of (31). In practice, a more precise computation must be made to determine the sign of C. For this purpose the theoretical coefficients are not sufficiently accurate.

The second-order equation of motion (17) is discussed from the physical viewpoint by Nomoto et al (1957), with particular reference to the use of a truncated approximation where the highest derivatives with respect to r and δ are deleted. This simplification is discussed further by Fujino and Daoud (1978).

Nonlinear corrections to the equations of motion are required to describe the steady-state turning maneuver, especially for ships which are unstable. For this purpose empirical terms are added to the equations of motion, typically involving combinations of odd powers of v,r and δ. It is argued that the coefficients of even powers in these variables are zero, based on symmetry of the hull about y=0. The existence of a cross-flow drag, proportional to the square of the lateral velocity, is a contradiction to this argument. A more complete nonlinear model should include terms proportional to $v|v|$, etc., in addition to the third-order terms such as v^3. Logically the latter are associated with nonlinearities in the potential flow, which are analytic in the body motions and can be expanded in Taylor series of conventional form. By comparison, the cross-flow drag due to separation is not analytic at v=0, although it can be expanded in separate power series for $v \gtrless 0$. For symmetric hulls, the ultimate result is a composite expansion, with terms such as $v|v|$.

MANEUVERING IN SHALLOW WATER

For the analysis of ship maneuvering in shallow water, the most important parameter is the ratio h/T, where h is the fluid depth and T the draft. If this ratio is substantially greater than one, say on the order of two or more, the only modification required in the slender-body approach is to correct the two-dimensional added-mass coefficient m' for the effect of finite depth. Simple estimates of this correction can be made if h/T is large compared to one, as described for a circular section by Lamb (1932).

The two-dimensional added-mass coefficient increases significantly for $h/T \to 1$, as emphasized by Kan and Hanaoka (1964). Systematic computations of m' for a family of rectangular sections by Flagg

and Newman (1971) can be summarized by the asymptotic approximation

$$m' = \frac{\rho B T^2}{h-t} + \frac{4}{\pi}\rho h^2\left\{1 - \log[4(1-T/h)] + \frac{1}{3}(1-T/h)^2\right\}$$
$$+ O(1-T/h)^3 \ . \qquad (32)$$

Here m' is the added mass of the submerged section, equal to half of the corresponding result for the double body. A more general asymptotic approximation derived by Taylor (1973) can be applied to sections with rounded corners, but the higher-order correction of (32) given by Taylor appears to be incorrect. Approximations based on (32) agree with exact computations within a few percent, for $h/T \lesssim 2.5$. An important exception is the case B=0, or a flat plate, where (32) should be replaced by the exact result

$$m' = -\frac{4}{\pi}\rho h^2 \log\cos(\pi T/2h) \ . \qquad (33)$$

The first term in (32) is inversely proportional to the underkeel clearance. Thus, in shallow water, the added mass increases dramatically, by an amount which is proportional to the beam. Typical results are shown in Figure 3.

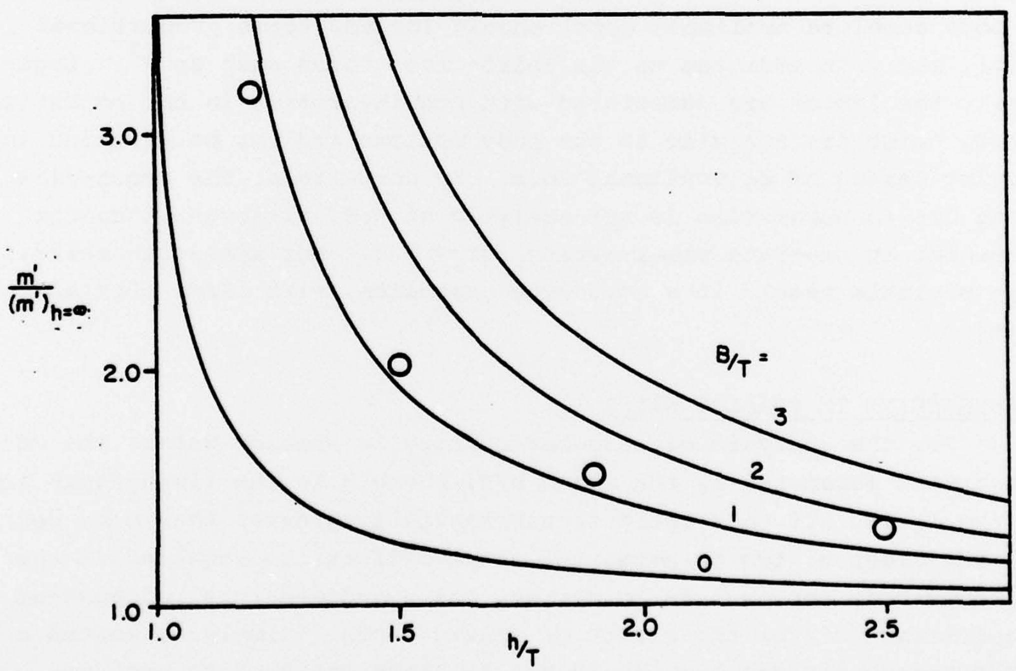

Figure 3. Dependence of added-mass coefficients on depth/draft ratio. The solid curves are theoretical values for two-dimensional rectangles, with beam/depth ratios as shown. The circles denote experimental data reported by Fujino (1976) for the Mariner model at a Froude number (based on length) of 0.1.

The two-dimensional added mass is unbounded as $h/T \to 1$, due to blockage of the flow beneath the keel. In the three-dimensional case the flow is diverted around the bow and stern, relieving the pressure difference between the two sides and reducing the lateral force. This effect is apparent from the experimental data shown in Figure 3 for the Mariner ($B/T = 3.03$). In relatively deep water these three-dimensional experiments are consistent with the two-dimensional theory, for a rectangle of beam-depth $B/T \cong 2$, as might be deduced from an estimate based upon (6). As the ratio h/T is decreased, however, the three-dimensional added mass increases more slowly, and ultimately approaches a finite value as $h/T \to 1$.

A generalization of slender-body theory has been developed by Newman (1969) to account for this phenomenon. The essential concept is to match an outer flow, predominantly two-dimensional in the horizontal plane, and an inner flow which is two-dimensional in the transverse plane. Matching of these two solutions leads to an integro-differential equation similar to that which governs the circulation in lifting-line theory. The resulting side force and moment tend in the deep-water limit to the conventional results of slender-body theory derived in the preceding section, and in the limit of zero clearance to the two-dimensional results of thin-wing theory. In the intermediate regime the problem is analogous to a "porous" wing, as noted in the Introduction. Tsakonas, et al (1977) present calculations based on this theory for several ship hulls, including the correction for Froude-number effects outlined by Breslin (1972).

Hess (1977) performed calculations for the steady force and moment due to sway and yaw, and extended the theory of Newman (1969) to include shallow-water effects on the rudder. Illustrative results are shown in Figure 4. The reduction in rudder moment in shallow water is particularly striking. This can be explained in terms of the limit $h/T=1$, where two-dimensional thin-wing theory is applicable. The appropriate wing has zero angle of attack, and zero camber except for the rudder or tail flap. The resultant center of pressure depends on the ratio of flap chord to total chord. As this ratio is decreased, the center of pressure moves aft from the quarter-chord point. As the flap chord tends to zero, the limiting value of the center of pressure is at the midchord position. With this model applied to the ship in the limit of zero underkeel clearance, the induced rudder force is centered amidships.

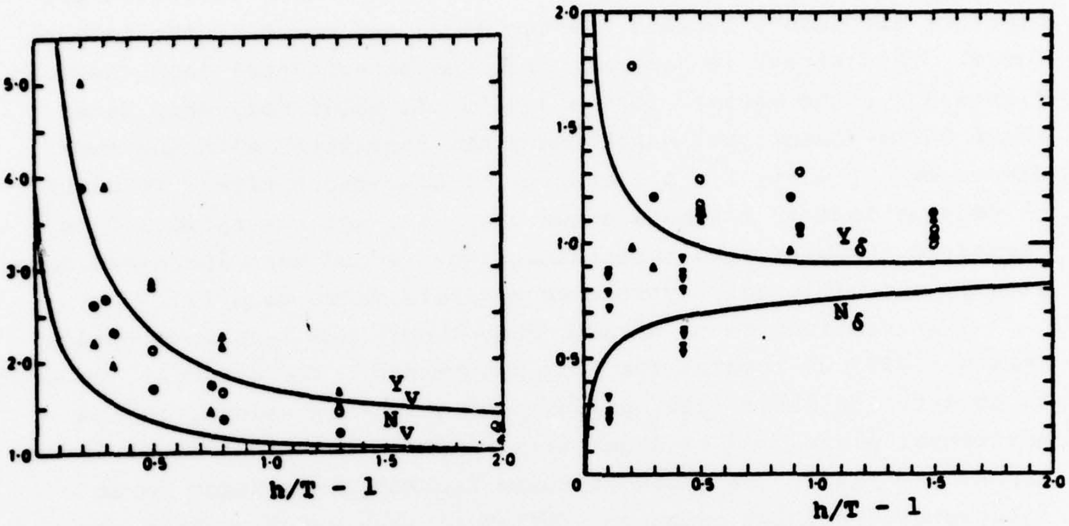

Figure 4. Effect of shallow water based on computations for a simplified hull form, from Hess (1977). Curves are ratios of the force or moment to the respective limits in deep water. Experimental data for the force are denoted by circles, and for the moment by triangles.

The calculations of rudder effectiveness by Hess (1977) are for steady-state conditions. Unsteady effects can be estimated for the limiting case h/T=1. We assume that the rudder chord is small compared to L, and that the rudder angle is a step-function in time. Neglecting added-mass effects at the instant of rudder deflection, the resultant force is proportional to the Wagner function, and increases monotonically from an initial value equal to half of the steady-state force. Since the steady-state moment about the midchord position is zero, and the moment about the quarter-chord point is constant (cf. Newman, 1977) it follows that the center of pressure is initially at the three-quarter-chord position, or midway between the midship station and stern. As time increases, the center of the rudder force moves forward from this point to the midship section, and the rudder moment is decreased.

Calculations of the force and moment in shallow water indicate that the limit of zero underkeel clearance is approached quite slowly. The results in Figure 4 show that the clearance must be less than half the draft for a 100% increase in the forces, and less than

10-20% of the draft for a comparable change in the moments. Thus the simple limits for h/T=1, which follow from two-dimensional thin-wing theory, are relevant only to indicate the mathematical limits for the more general calculations, based on the "porous" model which accounts for the cross flow in the gap beneath the keel. In effect, as emphasized by Tuck (1975), a relatively small gap is remarkably effective in relieving the pressure difference between the two sides of the ship.

RESTRICTED-WATER EFFECTS

The motion of a single vessel parallel to an adjacent vertical bank is related by the method of images to the parallel motion of two identical ships abreast. Each of these equivalent problems is a special case of more complicated situations, such as the motion of a single vessel near an irregular channel boundary, or the interaction between two ships moving with different velocities.

Here again, the fluid depth is an important parameter. The theoretical methods to be employed for deep or shallow water are fundamentally similar to the corresponding techniques described in the preceding sections for ship maneuvering in unrestricted water.

In deep water, the effects of other vessels or channel boundaries can be represented by an incident velocity field, with lateral component $V(x,t)$. Physically, this is analogous to a cross-current. Variations of this velocity field with respect to space are allowed, with the restriction that $V(x,t)$ does not change significantly over distances comparable to the beam or draft. Thus each section of the ship hull is affected by a locally constant cross current. The resultant force, due to the combined motions of the ship and this current, can be expressed as a generalization of (1) in the form

$$f(x,t) = -\left(\frac{\partial}{\partial t} - u\frac{\partial}{\partial x}\right)[(v+rx-V)m'] + \rho S\left(\frac{\partial V}{\partial t} - u\frac{\partial V}{\partial x}\right) . \qquad (34)$$

The first member of this equation is analogous to (1), with the relative velocity corrected to include the incident current. The second member of (34), proportional to the sectional area S, can be interpreted as a horizontal buoyancy force due to the pressure gradient of the incident velocity field.

Integration of (34) along the hull gives the total force and moment, including the additional contributions from the external field V. The principal task is to determine this incident velocity.

If the source of the external field is substantially distant from the ship's cross section, in terms of the characteristic scale

of the beam or draft, the effect of the slender ship on this external disturbance will be small, and the two can be superposed neglecting their interactions. For example, the velocity field due to a second ship can be calculated as if the first ship were absent.

This approach is applied by Tuck and Newman (1974) to the interaction between ships moving on parallel courses in deep water. When the two ships are abreast, or when one ship is moving parallel to a vertical bank, an attractive force acts in conjunction with a moment which is in the bow-out direction for conventional hull forms. The magnitude of this interaction decreases rapidly with increasing lateral separation. If the two ships are displaced longitudinally, the force and moment are oscillatory functions of this "stagger" distance. Comparison with experiments indicates reasonable qualitative agreement, but the theory apparently underpredicts the magnitude of the force by about 40%. The same approach is applied to two ships moving with different velocities, in the same or opposite directions. Generally speaking the interaction force and moment are largest on the slower of two such vessels.

For analogous problems in shallow water, a stronger interaction results from the horizontal constraint to the flow. In particular, the effect of the ship on the incident velocity field can not be neglected, regardless of the lateral separation. The simplest examples follow from the limit of zero underkeel clearance, where the resulting problem is similar to a two-dimensional thin wing in ground effect. For this relatively simple case, calculations of the steady interaction between two adjacent vessels are presented by Tuck and Newman (1974). Reasonable agreement is shown with respect to two-dimensional experiments. The corresponding problem for two ships with different speeds is solved by King (1977). In the latter case an unsteady vortex wake exists behind each body, leading to a more complicated time-dependent interaction.

The theory of Newman (1969) can be extended to analyse restricted water effects with a nonzero underkeel clearance. Once again a vortex wake exists behind each vessel if the interaction is unsteady. Solutions for two ships on parallel courses have been developed independently by Yeung (1978a) and Yung (1978). The interaction force and moment are similar to those for deep water, but with larger magnitude. Computations by Yeung for a head-on encounter of two identical ships are shown in Figure 5. Here, as in deep water, the largest force acts on the slower vessel.

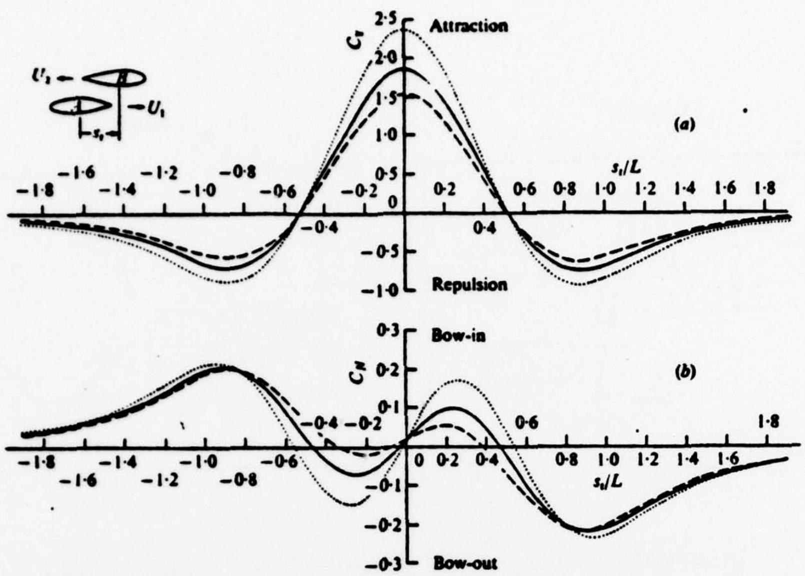

Figure 5. Sway force and yaw moment for two identical ships, passing in opposite directions in shallow water with T/h=0.9. The lateral separation distance is L/2. The force and moment are nondimensionalized by the factors $\frac{1}{2}\rho BTU_1U_2$ and $\frac{1}{2}\rho BTLU_1U_2$, respectively. The solid curve is for $U_1=U_2$. The broken curves are for $U_1/U_2=1.5$, with the larger force (••••••) corresponding to the slower ship.

The steady operation of a ship near a vertical canal wall, or parallel to a second vessel, requires a precise balance of the interaction force and moment by suitable combinations of yaw and rudder angles. An analysis of this configuration for shallow water is given by Hess (1978a).

If a ship approaches a bank at an angle, the resulting interaction is unsteady. This situation is analysed for shallow water by Yeung and Tan (1979). The most interesting result is that the ship is diverted by a bow-out moment and, at a late stage in its approach, by a repulsive force. This is confirmed by experiments with a free-running model conducted by Dand (1976). Yeung and Tan also consider unsteady interactions resulting from the shape of the bank. The example shown in Figure 6 is for a ship moving parallel to one side of a 90° bend. Note that the force and moment depend on the direction of the ship's passage, due principally to the interaction between the bank and the vortex wake. A similar analysis by Yeung (1978b) is applicable to the prediction of the lateral force and moment in a cross current.

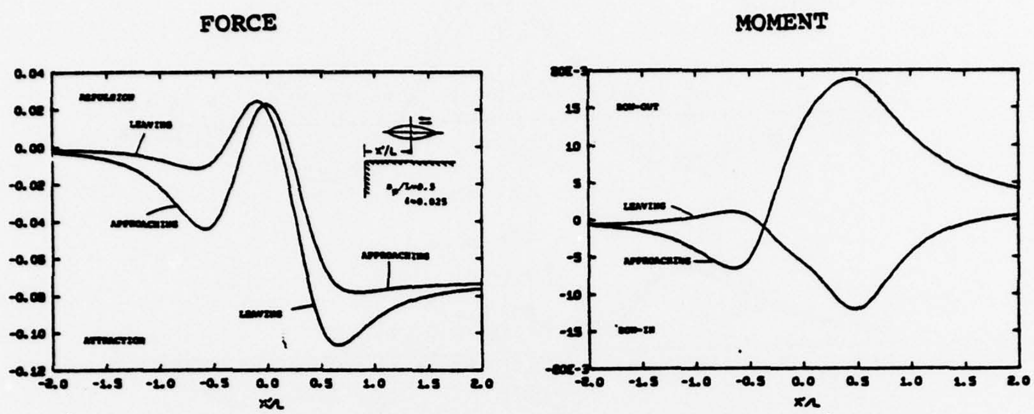

Figure 6. Interaction force and moment for a ship moving parallel to one side of a right-angle vertical wall at a separation distance of 0.5 L. In this example T/h=0.975. The force and moment are nondimensionalized with respect to the factors $\frac{1}{2}\rho u^2 BT$ and $\frac{1}{2}\rho u^2 BTL$, respectively.

A ship moving off the centerline of a canal experiences a lateral force and moment due to the resulting asymmetry. Typically the force is toward the nearest wall, and hence destabilising with respect to an intended track parallel to the canal. This problem is treated in the context of the shallow-water theory by Beck (1977). The illustrative results shown in Figure 7 are notable for their agreement with experiments.

Finally we consider a class of steady problems where the bottom is not flat and horizontal, but the depth contours are parallel to the ship's track. Numerical results for a flat "sloping beach" are given by King (1978), and for a "dredged channel" by Beck, Newman and Tuck (1975). In the latter problem the channel section is rectangular, and surrounded by regions of constant depth. The bottom topography is symmetrical about the ship's centerplane. Of particular interest in both studies is the existence of a transcritical depth separating regions of sub- and supercritical Froude number. This is an intriguing mathematical problem, but the calculations do not reveal important effects unless the entire shallow region is transcritical.

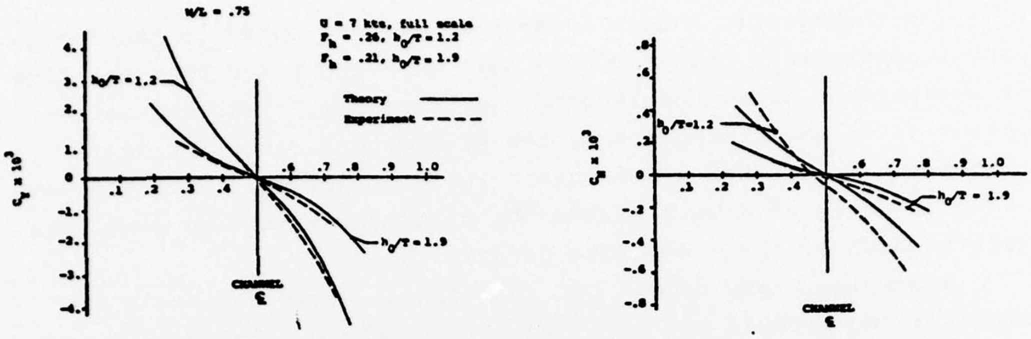

Figure 7. Lateral force and yaw moment on a tanker due to offcenter position in a canal, from Beck (1977). The canal width is 0.75L, and the abscissa is the ratio of distance off the canal centerline divided by the canal width. The force and moment are nondimensionalized by the factors $\frac{1}{2}\rho u^2L^2$ and $\frac{1}{2}\rho u^2L^3$ respectively.

CONCLUDING REMARKS

Considerable progress has been made in recent years to provide theoretical predictions of ship maneuvering characteristics. This progress is evident especially in the study of shallow and restricted water effects, where empirical information is limited and many new problems have been raised by contemporary shipping practice.

Existing comparisons between theory and experiments are less satisfactory than might be desired. In most cases the degree of qualitative agreement is reasonable, and the theoretical descriptions are useful in the context of interpolation between sparse experimental and empirical data. But from the quantitative standpoint, differences as large as 50% are common. It is likely that most of these differences are the fault of the theories, rather than experiments.

The principal defects of the theory are overprediction of the force and moment due to the rudder, and underprediction of the force on the hull due to its own lateral motions or external disturbances. Attempts to improve the prediction of rudder effectiveness by semi-empirical corrections have not been sufficiently rewarding. Indeed, it seems likely that the flow at a ship's stern is too complex to describe adequately by any conceivable theory.

On the other hand, a more optimistic view seems justified for the force acting along the hull.

The theoretical models described here are based on the fundamental assumptions that the ship hull is slender and that the flow is unseparated along the length. In principle, these can be related to two small parameters, the slenderness ratio (or draft-length ratio) and the hydrodynamic angle of attack. Strictly, the ratio of angle of attack divided by slenderness must be small, as well as each of these separate parameters.

A complementary domain can be defined, where the angle of attack is comparable with the slenderness ratio. The Bollay model for a planar wing of small aspect ratio is intended as a solution in this domain, and has been applied to ship maneuvering problems by Inoue (1956). Further work in this direction seems warranted, to develop a nonlinear slender-body theory consistent with the classical approach, for sufficiently small angles of attack, and similar to the Bollay model in the nonlinear regime. This hypothetical solution should be applicable to a body of ship-like form, with a prescribed point of separation at the bilge.

Much thought was given to the latter problem by the brilliant ship hydrodynamicist Roger Brard. His David W. Taylor Lectures (Brard, 1976) are a promising point of departure for future work in this field. It is appropriate here to quote the philosophy with which Brard approached this subject:

> *"To conclude these lectures, I should like to say that my firm opinion is that neither pure empiricism alone, nor pure theory alone can provide us with means sufficient for resolving all the problems relevant to ship maneuverability. But I am convinced that by making a judicious use of experiments and of theoretical schemes, many points still rather obscure will be gradually removed."*

ACKNOWLEDGEMENT

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